

## Effect of a stepped piston on an alpha Stirling engine

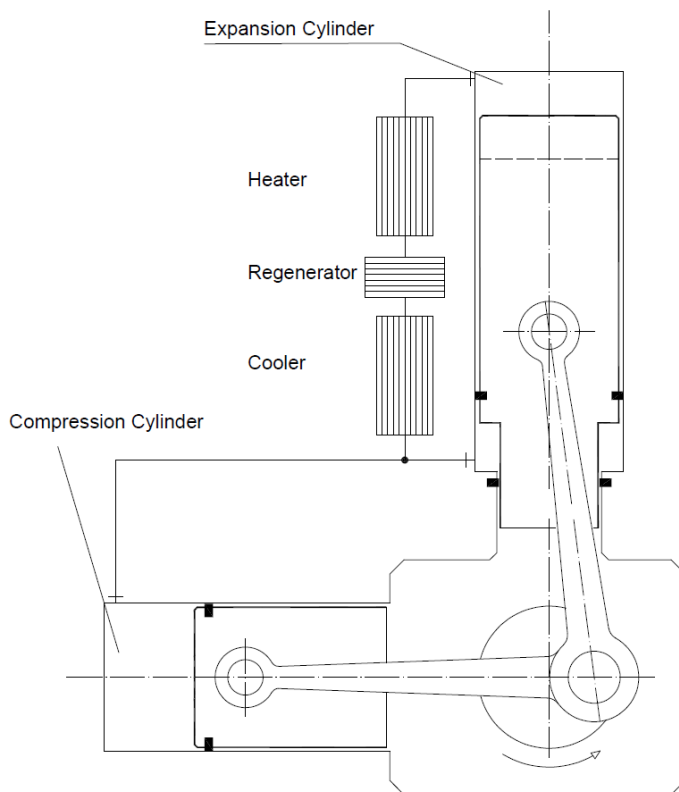
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This report is about an investigation into the effect of a stepped expansion piston on an alpha Stirling engine where one piston face on the cold side is connected to the pressurised crank case, the remaining annular piston area on the cold side is connected to the compression space (Figure 1). At first glance, this modification only introduces a volumetric phase shift on the compression side with respect to the expansion side; however, there is more to this than meets the eye as will be described in this report.

### Simulation software Sage

For this investigation, the physical dimensions and measured performance data of an experimental Frauscher Stirling engine prototype were used. In the following, this engine is

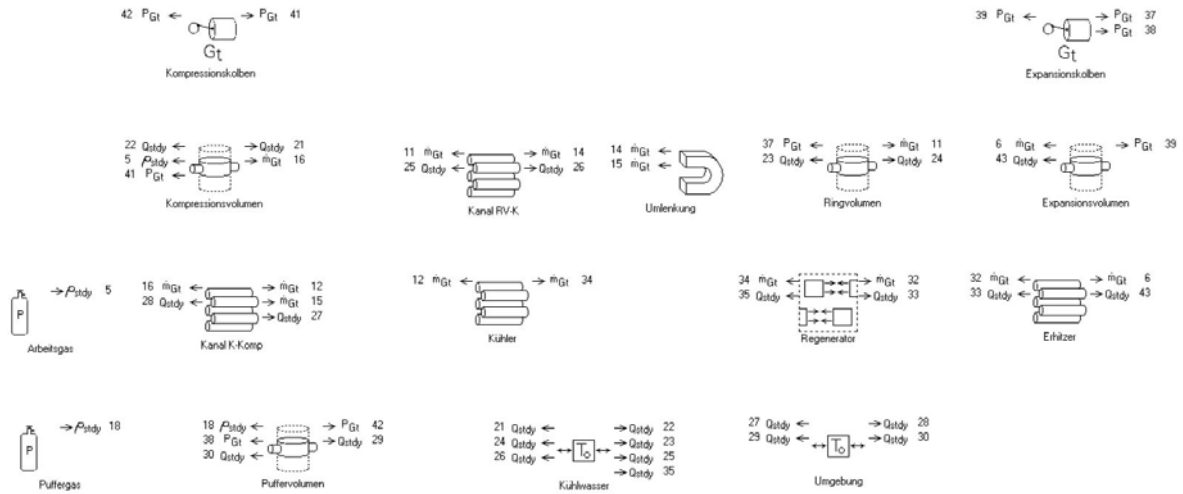
referred to as an *AlphaGamma* Stirling engine. The key parameters included the piston strokes as well as the mean pressure the engine is operated at. Based on this information, a model was created in the commercial Stirling simulation software Sage written by David Gedeon<sup>1</sup>.



**Figure 1:** Schematic of the Frauscher AlphaGamma Stirling engine (provided by Frauscher)

In its ability to model heat transfer processes and fluid friction of oscillating flow – even through porous media such as the regenerator matrix – Sage can be regarded as a very powerful 3<sup>rd</sup> order model. Each component of the system is analysed in a one-dimensional form, subdivided into a user-defined number of spatial increments. While Sage primarily models the thermodynamic behaviour of the working gas, it also takes into account the interaction of the gas with the adjacent walls, as well as heat transfer by conduction in the

<sup>1</sup> Gedeon, D., Sage User's Guide. Gedeon Associates, Athens, OH, 2013



**Figure 2:** Screenshot of the Sage simulation model of the Frauscher AlphaGamma Stirling engine

canister walls in both axial and radial directions. In this respect, Sage is somewhere between a one- and a two-dimensional modelling tool. Sage operates in a frequency domain and solves all fundamental conservation equations for each cell until it converges to a solution. Figure 2 depicts what the modelled *AlphaGamma* configuration looks like in Sage.

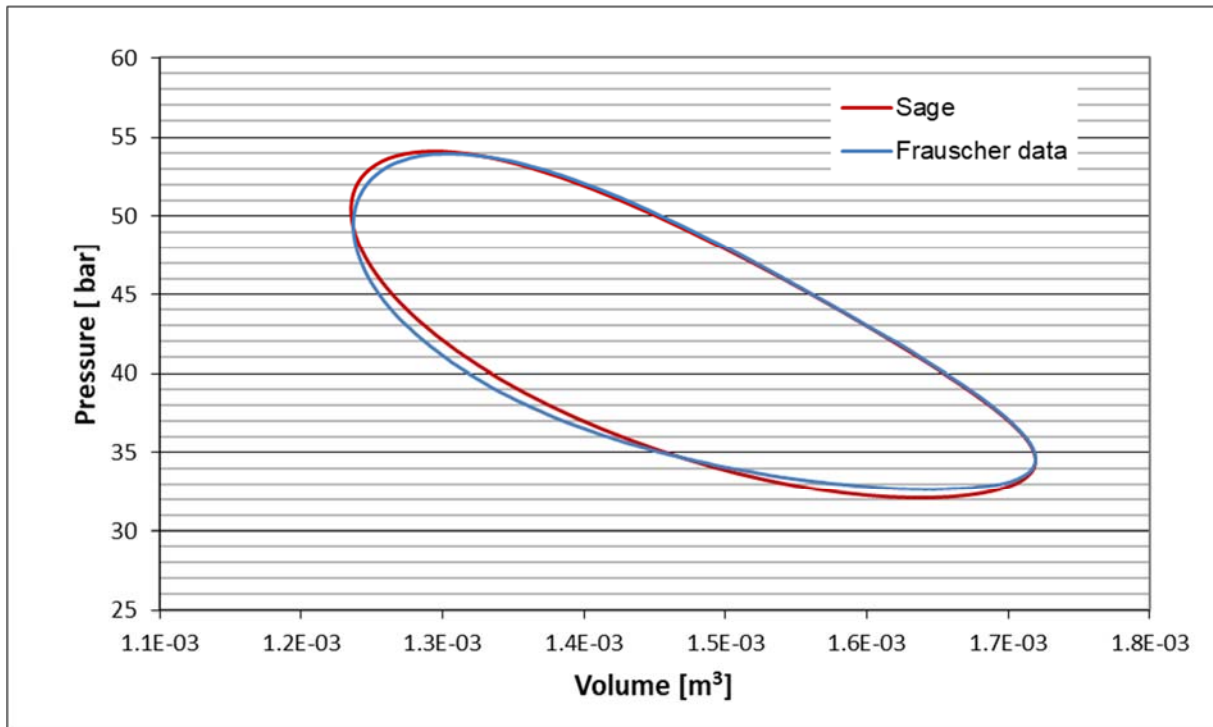
### Validation of the Sage simulation

In order to validate the Sage model, a comparison was made between the experimental  $p$ - $V$  data provided by Frauscher and the Sage simulation (Figure 3). A few assumptions had to be made for the simulation:

- Since the minimum and maximum volumes are kinematic constraints independent of any simulation, and since there is inevitable inaccuracy in the determination of the actual gas volume in a real engine, the volume was adjusted in Sage by adding a small amount of dead volume in the regenerator canister.
- Another minor adjustment was required for the average gas pressure. While Sage keeps the average gas pressure as specified by the user constant by adjusting the gas mass, in a real engine, the average gas pressure slightly increases after start-up.
- The volume for the buffer space was chosen such that that the pressure amplitude matched the measured one.
- Other than that, only the heat source and heat sink temperatures were chosen to be the same as in the experiment.

As can be seen in Figure 3, the measured  $p$ - $V$  data by Frauscher and the simulation results obtained with Sage agree very well. Sage underestimates the  $p$ - $V$  area by only 6%. It is unclear, however, how large the experimental inaccuracy is. It should be noted that the agreement between simulation and real prototype is almost ‘spot-on’ during the expansion phase (top of the  $p$ - $V$  loop), while there is more discrepancy during the compression phase (bottom of the  $p$ - $V$  loop). An explanation as to why this is cannot be given at this stage.

Furthermore, Sage predicts a net power output of 8864 W at a speed of 1007 rpm that the prototype was tested at. Frauscher measured an electrical power output of 6674 W, which



**Figure 3:** Comparison between the experimental  $p$ - $V$  loop and Sage results

would indicate that approx. 25% of the indicated power is lost due to mechanical friction and in the mechanical to electrical conversion process.

Another encouraging validation of the Sage simulation is the comparison with some temperature measurements. Table 1 shows the measured temperatures in three locations and the corresponding Sage results. With the exception of the hot gas temperature at the inlet to the expansion cylinder where the difference is approx. 15 K, the remaining gas temperatures are almost identical.

**Table 1:** Comparison of gas temperatures between the experiment and Sage

Location	Experiment	Sage
Hot gas temperature at inlet to expansion cylinder	700.1°C	684.8°C
Hot gas temperature at inlet to regenerator	724.4°C	724.3°C
Cold gas temperature at inlet to compression cylinder	50.5°C	50.7°C

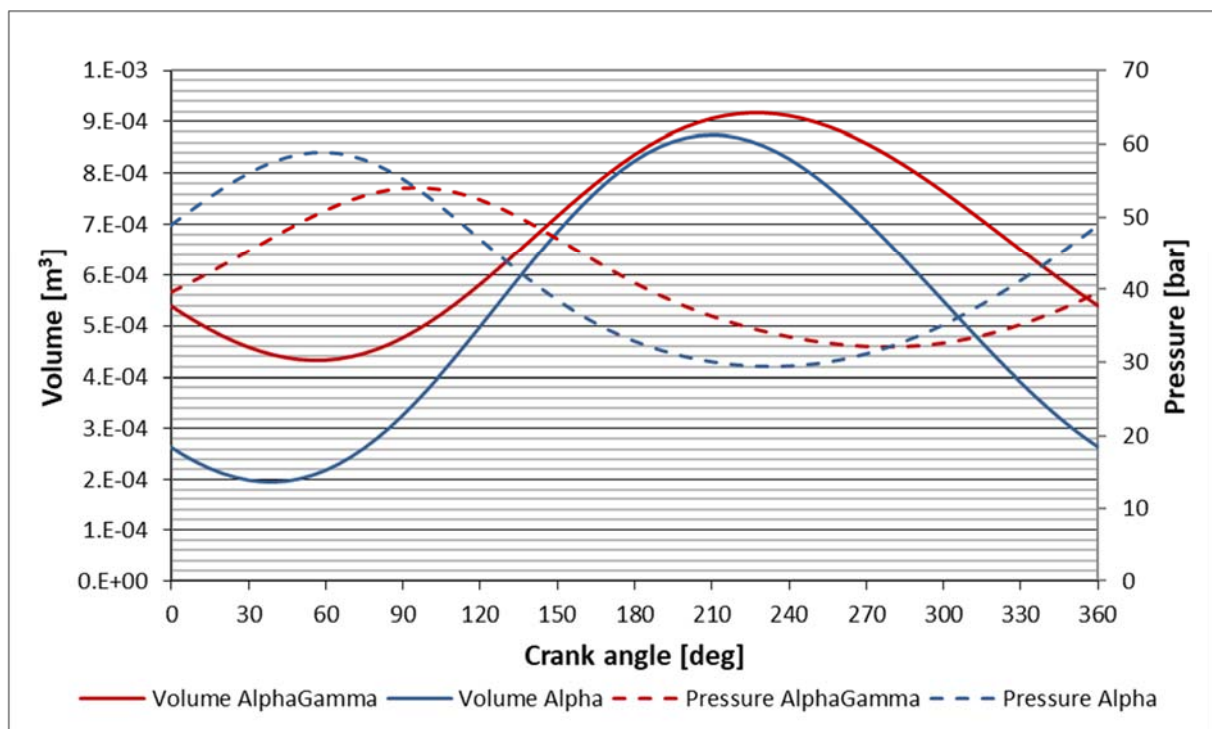
## Volumetric phase relationship

Probably the most obvious effect the proposed configuration has on the thermodynamic behaviour compared to a pure alpha configuration is on the volumetric phase relationship between expansion and compression space. Since the compression space in the *AlphaGamma* engine is now connected to two piston faces that are moving out of phase by  $90^\circ$ , the combined effect changes the phase difference between the total volume and the pressure, which is critical for the  $p$ - $V$  work in terms of the integration of  $p dV$  over a complete cycle.

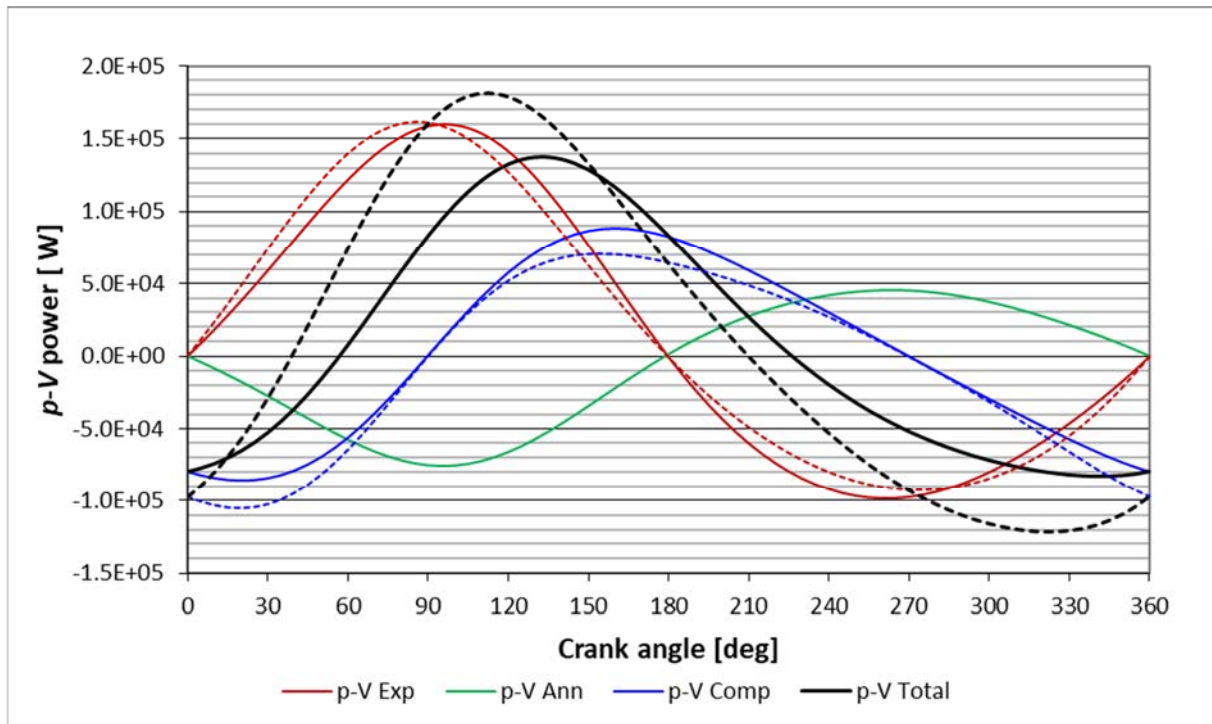
Figure 4 shows the cyclic change of the total volume and the pressure over a complete cycle for both the alpha configuration (blue) and the *AlphaGamma* engine (red). In both cases, the pressure slightly lags behind the minimum volume. By inspection of the numerical values in Excel, it was found that the pressure lags by only  $20^\circ$  in the alpha configuration, while it lags by about  $40^\circ$  in the *AlphaGamma* engine. What effect this has on the work done on and by the gas on the pistons can be seen in the following  $p$ - $V$  power diagrams.

## $p$ - $V$ power

Figure 5 shows the cyclic  $p$ - $V$  power of the working gas in the expansion space (red), the annular space created by the stepped piston (green), the compression space (blue), as well as the total  $p$ - $V$  power of the entire gas space. The diagram shows a comparison between the alpha configuration (dashed lines) and the *AlphaGamma* (solid lines). It should be noted that the Sage model of the alpha configuration was created by simply removing the annular gas space (by removing the step in the piston) and the duct connecting the annular space with the compression space in the *AlphaGamma* Sage model. As a result, the alpha engine has approx. 8% less dead volume and an almost 40% higher pressure amplitude. However, the alpha engine produces a net power of only 8592 W, while the *AlphaGamma* generates 8868 W.



**Figure 4:** Comparison of total volume and pressure between *AlphaGamma* (red) and *Alpha* (blue)



**Figure 5:** Cyclic  $p$ - $V$  power done by the gas in various space for both the alpha configuration (dashed lines) and the AlphaGamma (solid lines)

According to the Engineering Thermodynamics sign convention, a positive value in Figure 5 means that the gas performs work on its boundary. Since the expansion piston is at top dead centre for a crank angle of  $0^\circ$ , the  $p$ - $V$  power has just passed zero and increases in positive direction as the piston moves down and the expansion space becomes larger (red lines). The opposite is true for the annular gas space in the *AlphaGamma* (green line) as the two piston faces are out of phase by  $180^\circ$ .

While the cyclic  $p$ - $V$  power of the expansion space is very similar for both the alpha engine and the *AlphaGamma*, the  $p$ - $V$  power of the compression space is clearly shifted in positive (i.e. power-producing) direction in the *AlphaGamma*. This manifests itself in the compression piston in the *AlphaGamma* requiring no work input, in fact, even producing a small amount of power output as can be seen in Table 2 (highlighted in bold). In this respect, the compression piston almost behaves like a displacer as in a gamma configuration.

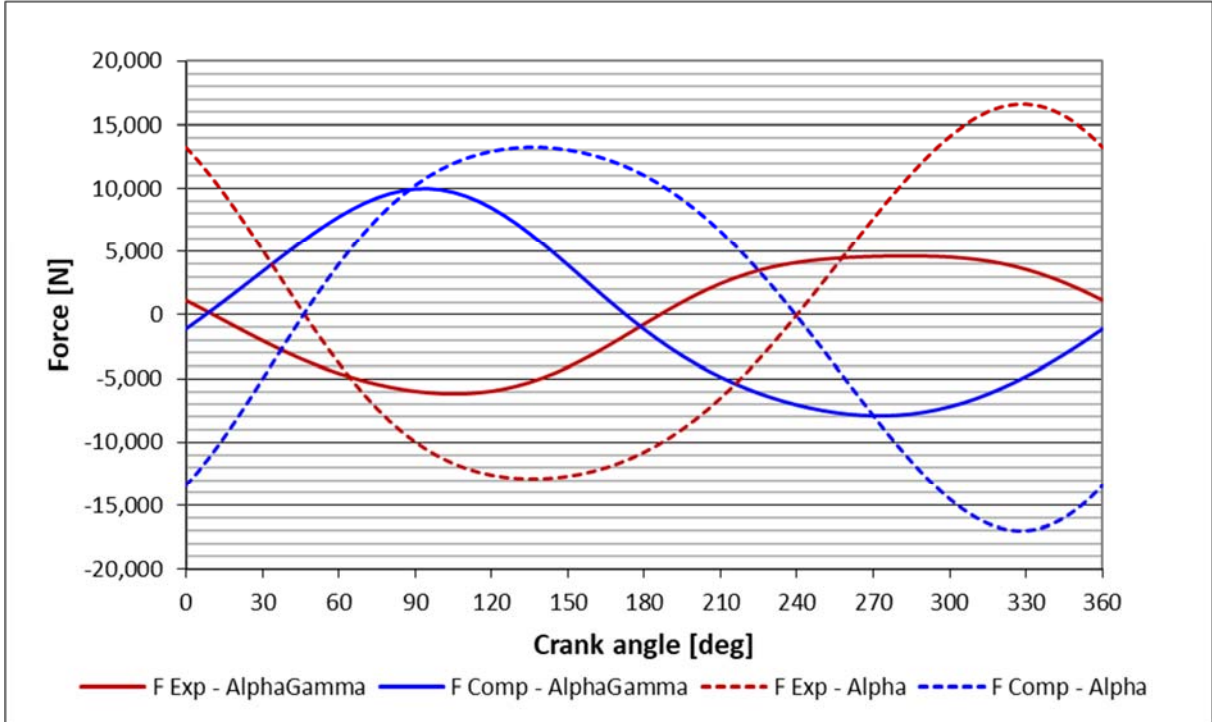
**Table 2:** Net power of compression and expansion piston in both the alpha and the AlphaGamma engine

	Alpha	AlphaGamma
Net power compression piston	-9649 W	<b>138 W</b>
Net power expansion piston	18240 W	8730 W
Net power produced by the engine	8591 W	8868 W

The values in Table 2 clearly show that the expansion piston in the *AlphaGamma* engine does both the expansion and compression work on the gas, resulting in less than half the net power as in the alpha configuration. Consequently, this results in much lower piston, and thus, bearing forces as will be discussed below.

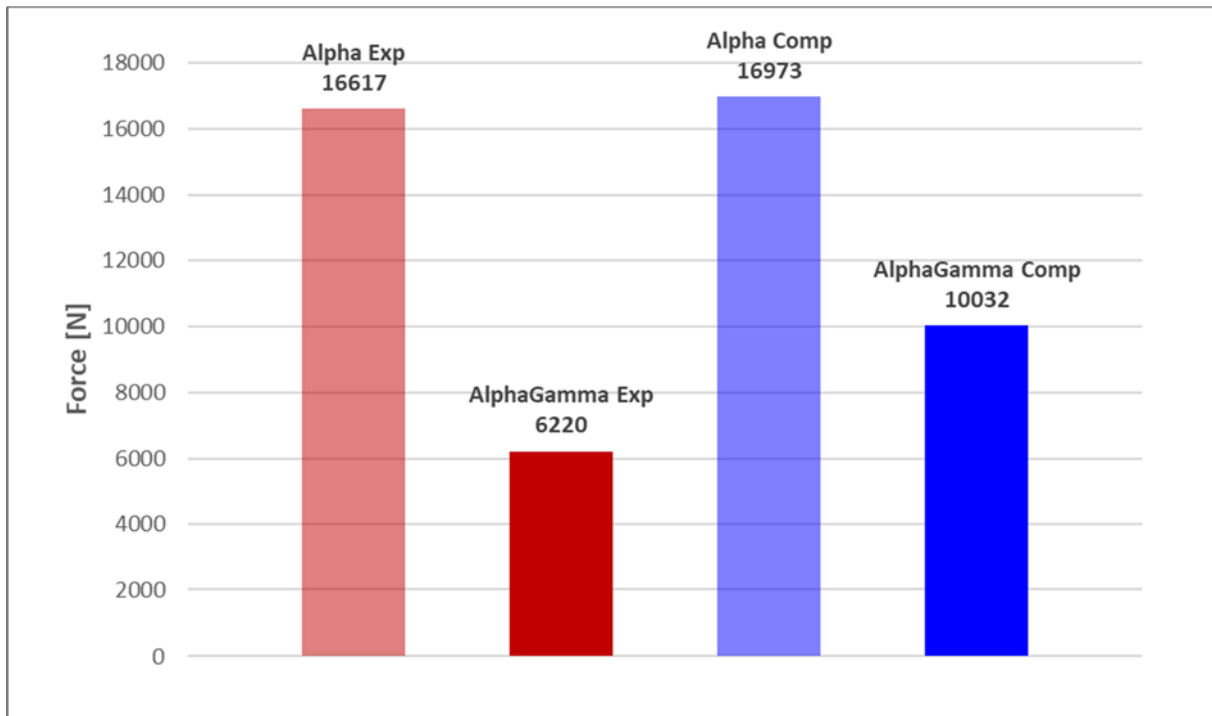
**Piston forces**

Figure 6 shows the cyclic ‘required forcing function’ for the compression and expansion piston for both the alpha configuration as well as the *AlphaGamma* engine. In Sage, the ‘required forcing function’ is the force that is required to execute the prescribed motion, taking into account inertia forces due to acceleration as well as the instantaneous gas forces. Based on the previous discussion of the cyclic *p-V* power in various gas spaces of both engines, it is not surprising that the force amplitudes are much smaller in the *AlphaGamma* engine.



**Figure 6:** ‘Required forcing function’ for the compression and expansion piston in both the alpha configuration and the *AlphaGamma* engine

Figure 7 shows the maximum occurring piston forces from Figure 6 for the expansion and compression pistons in both the Alpha and the *AlphaGamma* engine. The force on the compression piston of the alpha configuration is approx. 1.7 times higher than in the *AlphaGamma* engine. The difference in the expansion piston is even higher with an almost three times higher force in the alpha configuration compared to the *AlphaGamma* engine. This means that there is a significantly smaller load on the bearings in the proposed *AlphaGamma* configuration. The reduced forces on the piston also mean that the side-loads on the piston seals are much lower, which results in less friction and less wear of the seals.



**Figure 7:** Maximum forces on the expansion (red) and compression (blue) pistons in both the Alpha and the AlphaGamma Stirling engine

### Discussion and conclusion

A simulation of the proposed *AlphaGamma* engine in Sage reveals that the piston forces can be substantially reduced by using a stepped expansion piston in an alpha configuration and connecting the annular gas space to the compression space. By executing the Stirling cycle between three piston faces that are all out of phase, a phase shift between the overall gas volume and the pressure can be achieved. In this particular case, this leads to an almost zero net  $p$ - $V$  power done by the compression piston (which makes it act as a displacer), and to the opposing gas forces on the expansion piston resulting in a much smaller net force.

While this study clearly illustrates the difference between a traditional alpha configuration and the proposed *AlphaGamma* engine, it should be noted that a fair and unbiased comparison between the two configurations is very difficult to make. Leaving everything the same, by removing the additional duct and annular gas space from the *AlphaGamma* in order to turn it into an alpha engine, the overall volume changes. This leads to a higher pressure amplitude in the alpha configuration, with the further consequence of a slightly reduced efficiency.

Making a comparison based on a similar net power output, however, the mean pressure in the alpha configuration would have to be slightly increased in order to compensate for the somewhat lower power output. Again, this would have an effect on the pressure amplitude. On the other hand, making the pressure amplitude the same by reducing the piston stroke or adding more dead volume in the alpha engine would reduce the power output even more. It was therefore decided to adjust as little as possible, i.e. remove the ducts and keep the mean pressure the same. That way, the power output of the alpha configuration is only 3% lower than of the *AlphaGamma* engine, and a reasonable comparison can be made.

In conclusion, the proposed *AlphaGamma* engine is a very clever modification of the traditional alpha configuration that reduces the forces on the compression and expansion piston quite substantially. It would be interesting to carry out a study of the newly introduced parameters, such as the diameter ratio of the stepped piston or other phase relationships, to see what their effect is on power output and efficiency.

A handwritten signature in blue ink that reads "Michael Gschwendtner". The signature is written in a cursive, flowing style.

Dr. Michael Gschwendtner

Auckland, December 2021